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PHYSICO-MATHEMATICAL MODEL OF THE LIFE CYCLE OF A SINGLE-CYCLE TWO-STROKE GAS ENGINE OF THE BROWN TYPE OPERATING ON HYDROGEN TECHNOLOGY

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Abstract. In this work, physico-mathematical models describing some key issues of the operation of a single-cycle two-stroke gas/stream Brown engine operating on hydrogen technology are developed and studied. The main issues that are discussed in this work are: constructing mathematical models, conducting numerical experiments; clarification of the conditions under which a gas engine operating on hydrogen technology will complete at least one full life cycle.

Keywords: *Brown type gas engine, hydrogen technology, engine life cycle, mathematical model, weights of gas-dynamic parameters.*

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1. Introduction

Despite the fact that only in the last 20-30 years concern for preservation of the environment has become a global mass problem of mankind, questions about changing the standard internal combustion engine have been raised by scientists and inventors regularly since the beginning of the 19th century (Cummins, 2018). For some reason, it is generally accepted that hydrogen energetics appeared quite recently: this is due to the fact that hydrogen energy has not found wide application yet, although thousands of scientists have been working on the problem of mastering one of the main elements of the periodic table for a very long time.

Thus, in 1806 French inventor François Isaac de Rivaz (1752–1828) developed an engine powered by a mixture of oxygen and hydrogen obtained by water electrolysis.

In 1820, William Cecil, a researcher at Magdalene College of the University of Cambridge, published an article titled "*On the Application of Hydrogen Gas to Produce Moving Power in Machinery*", in which he described a model engine (now called Cecil's hydrogen engine) which he had constructed to operate using hydrogen according to the explosion-vacuum principle: atmospheric pressure drives a piston back against a vacuum to produce power and the vacuum itself is created by burning a hydrogen-air mixture, allowing it to expand and then cool. The Cecil's hydrogen engine had 3 ft diameter flywheel and ran at 60 rpm. In the same year, William Cecil, speaking at the Philosophical Society of Cambridge, called his engine "ardent spirit" and the mixture – turpentine and vapour of oil, suggested using hydrogen for driving machines for various purposes. Although the Cecil's hydrogen engine ran satisfactorily, vacuum engines never became practical.

In 1823-1833, Samuel Brown patented the first internal combustion gas vacuum engine to be applied industrially: the design used atmospheric pressure and was demonstrated in a carriage and a boat, and in 1830 it was in use commercially to pump water to the upper level of the Croydon Canal.

The main difference between a hydrogen device and traditional engines is the method of supplying the fuel liquid and subsequent ignition of the working mixture: at the same time, the principle of transforming the reciprocating movements of the crank mechanism into useful work remains unchanged (given that the combustion of petroleum fuel occurs rather slowly, the fuel-air mixture fills the combustion chamber before the piston reaches its extreme upper position - the so-called top dead centre). Then the rapid reaction of hydrogen makes it possible to move the injection time closer to the moment when the piston begins to return to bottom dead centre (it must be emphasized here that the pressure in the fuel system will not necessarily be high). If ideal operating conditions are created for a hydrogen engine, then it can have a closed-type fuel supply system, when the mixture formation process takes place without participation of atmospheric air flows. In this case, after the compression stroke, water vapour remains in the combustion chamber, which, passing through the radiator, condenses and turns back into ordinary water. However, the use of this type of device is possible only when the vehicle has an electrolyzer that separates hydrogen from water for its repeated reaction with oxygen. At the moment, achieving such results is extremely difficult. For stable operation of engines, engine oil is used and its evaporation is part of the exhaust gases. Therefore, a trouble-free launch of the power device and its stable operation on explosive gas without the use of atmospheric air is still an impossible task. There are two variants of hydrogen transport installations: units operating on the basis of hydrogen fuel cells and hydrogen internal combustion engines. The principle of operation of fuel cells is based on physical and chemical reactions. In fact, these are the same lead batteries, however the energy conversion efficiency of a fuel cell is slightly higher than that of lead batteries and is about 45% or even more.

At the beginning of May 2021, the agreement No. VP-L-2021/46 (from now on we will use SIC-VP-L-2021/46 for referring to this agreement) was concluded between the authors of this work and the Latvian Investment and Development Agency (in Latvian: Latvijas Investīciju un attīstības aģentūra – LIAA) for the period of one year, within the framework of which the authors conducted research on the following problems:

(a) Physical and mathematical modelling of a single-cycle two-stroke gas-steam internal combustion engine of the Brown type operating on hydrogen technology;

(b) Theoretical numerical study of the constructed physical and mathematical models;

(c) Analysis and processing of the obtained numerical calculations;

(d) Preparation of the theoretical basis for creating a prototype of such an engine;

(e) Technical and economic feasibility study of serial production and implementation of such an engine.

While implementing the above-mentioned agreement SIC-VP-L-2021/46, the authors of this work faced many complex scientific questions, in particular, the issue of calculating the dependence of gas interaction parameters and power; the question of calculating one full cycle of energy exchange as well as both the energy gain from the working substance and the energy loss so that the system reaches its initial state; the question of determining the remaining amount of energy required both for the electrolysis process for the next life cycle of the engine and for performing the intended engine operations (without load or with load); etc.

In the course of the implementation of SIC-VP-L-2021/46 in general, and particularly in the course of constructing a physical and mathematical model of the operation of a single-cycle two-stroke gas engine of the Brown type operating on hydrogen technology, we obtained some purely theoretical results: differential equations; functional equations that establish the relationship between controlled and controlling engine parameters; asymptotic formulas between some controlled parameters of a power plant; etc. Then, in the course of completing the tasks of SIC-VP-L-2021/46 in general, including while performing the task of constructing a physical and mathematical model of the operation of a Brown-type gas engine, it became clear to us that the relations between the main parameters (we established that for the Brown type engine we are studying, it is quite possible to limit ourselves to 30 parameters, which are the main ones), which characterize at least one full cycle of stable operation of a single-cycle two-stroke gas engine of the Brown type, are so complex and mutually influencing that even minor deviations of the reference/theoretical values for even only a few of these parameters lead to a very wide variety of situations during engine operation. Three of these quite possible situations are, in our opinion, the following limiting situations (LS):

LS-1. The limiting situation (a bad situation!) when the time between turning the power plant on (using a spark ignition system) and its spontaneous extinction can only be a few seconds. Naturally, in this situation, we cannot talk about any full cycle of operation of a Brown type gas engine!

LS-2. The limiting situation (a bad situation!) when after switching the power plant on, it operates in normal mode for some time, however spontaneous (in the sense of being uncontrolled) diffusion detonation of the hydrogen-air mixture can occur. It is obvious that, without scientifically based levers to prevent the above-mentioned spontaneous detonation of the energetic fuel (i.e., a hydrogen-air mixture) of a Brown-type engine, it makes no sense to develop such an uncontrollable/dangerous engine for its actual implementation/use.

LS-3. The limiting situation (a good situation!) when after switching on, the power plant in normal mode completes a full cycle with a residual energy reserve of slightly more than 18% (18.333(3)% and provided that the power plant operates “cleanly” without any external/additional peripheral cargo). Obviously, this situation, although being a good situation, does not allow the real use of a single-cycle two-stroke Brown type engine with a hydrogen-air mixture.

There are many reasons for the occurrence of the above-described limiting situations and a comprehensive study of all these reasons within the framework of a year-long low-budget project, such as SIC-VP-L-2021/46, turned out to be impossible for us: in our opinion, at least 2-3 years are needed and with the condition that a Brown type gas engine is built, tested and modified many times. However, as part of the implementation of SIC-VP-L-2021/46, we identified the main cause of occurrence of limiting situations LS-1 – LS-3, however, in this work, we will discuss neither this cause, nor its nature, nor these limiting situations themselves: the authors intend to prepare a new scientific article in the near future, which will be based on this work and which will discuss in detail all these and other scientific and technical issues that arose during the implementation of SIC-VP-L-2021/46 and some of which we have solved.

In conclusion of this section, the authors of this work would like to emphasize that the research carried out within the framework of SIC-VP-L-2021/46 and the theoretical results obtained far exceed the research tasks and required results fixed in SIC-VP-L-2021/46. Naturally, when we began research within the framework of SIC-VP-L-2021/46, we could not have predicted in advance how far we would progress in research. In the course of the research, we came to the conclusion that if there appear full-fledged local (Latvian) or international projects with a reasonable term (at least 2-3 years) and with a sufficient budget (at least 2-3 million euros), our research group will be able to create not only a prototype of a single-cycle two-stroke gas engine of the Brown type, but also a full-fledged low-power engine for implementation and use in low-power household and business equipment, such as vacuum cleaners, lawn mowers, low-power hydraulic pumps used in households, etc. If there were such a full-fledged project, then within its framework our research group could:

- Completely finish all the necessary theoretical research (this implies physical, chemical, mathematical, economic and environmental research);
- Commercialize the technology;
- Set up the production;
- Solve the problem of introducing this technology into domestic and foreign markets.

2. Mathematical modelling: the life cycle of a single-cycle two-stroke gas engine; optimal composition of the hydrogen-air mixture; weight coefficients of kinetic and dynamic parameters of the basic functional parts of the hydrogen power device; mutual influence of parameters on the life cycle of a single-cycle two-stroke gas engine; processes of ignition and combustion of a hydrogen-air mixture

In order to determine the optimal amount of hydrogen fuel (Remark 1) required for the life cycle of a single-cycle two-stroke gas engine, first of all, it is necessary to know the energy characteristics of hydrogen fuel. Due to the fact that nowadays in order to fuel hydrogen transport power devices one uses hydrocarbon products, such as H_2 – hydrogen gas, $H_2(-253°C)$ – liquid hydrogen, CH_4 – methane gas, $CH_4(-160°C)$ – liquid methane, lower alcohols (CH_3OH – methanol, C_2H_5OH – ethanol), CH_3OCH_3 – liquefied methoxymethane (radical-functional nomenclature name: liquefied dimethyl ether), $\{C_3H_8, C_4H_{10}\}$ – liquefied propane-butane, which is a mixture of liquefied saturated hydrocarbons C_3H_8 – propane and C_4H_{10} – butane, we present the energy characteristics of all these fuel sources in Table 1. In addition, the last line of Table 1 shows, for comparison, the energy characteristics of the traditional fuel source – gasoline $C_nH_{2\cdot n+1}$, $n = \overline{7,10}$.

***Remark 1**. Hydrogen fuel can be obtained by conversion of natural gas, previously purified from sulphur (usually ZnO is used or zeolites), on a nickel catalyst at a temperature 850÷1000 ºC. The resulting gas mixture must be separated – carbon dioxide is removed by washing the gas mixture with high pressure water and then by absorption with alkali solutions and finally, the hydrogen released in this process is purified (Fischer-Tropsch process):*

$$\begin{cases} CH_4 + H_2O \to CO + 3H_2 - 0.206\, MJ/mol; \\ CO + H_2O \to CO_2 + H_2 + 0.041\, MJ/mol. \end{cases} \Rightarrow$$

$$\Rightarrow CH_4 + 2H_2O \to 4H_2 + CO_2 - 0.165\, MJ/mol.$$

In addition, hydrogen can be obtained from methanol (the mass index among the accumulation of hydrogen in the form of liquid methanol is $\approx 8.5 kg/kg\ H_2$*) either by its dissociation* $CH_3OH \leftrightarrow CO + 2H_2 - 0.09\, MJ/mol$*, or by steam reforming* $CH_3OH + H_2O \leftrightarrow CO_2 + 3H_2 - 0.049\, MJ/mol$.

As can be seen from Table 1, according to complex criteria, the most appropriate is the use of methanol and hydrogen as a fuel for hydrogen transport power devices. However, it should be noted that hydrogen, as a fuel source, has a big drawback (almost the only drawback!) – the complexity of compact storage (mainly from an economic point of view, but technical complexity can also be added). However, in recent years, this disadvantage, as well as some minor disadvantages, such as the lack of a wide production, filling infrastructure, high cost, etc. limitations are successfully overcome step by step. Therefore, it can be said with a high degree of certainty that all these shortcomings are temporary factors hindering the widespread development of transport hydrogen energetics.

Table 1. Main energy characteristics of fuels used in hydrogen transport power devices.

Hydrogen fuel source	Hydrogen level, % of mass	Specific mass energy, *MJ/kg*	Specific volume energy, *MJ/litre*
$H_2,\ p = 35\, MPa$	100	120	3.6
$H_2,\ p = 100\, MPa$			11
$H_2(-253°C)$	100	120	8.4
$CH_4,\ p = 35\, MPa$	25	50	12
$CH_4,\ p = 100\, MPa$			35
$CH_4(-160°C)$	25	50	35
CH_3OH	12.5	20	16
C_2H_5OH	12	27	22
CH_3OCH_3	13	28.8	19.2
$\{C_3H_8, C_4H_{10}\}$	15÷18	46÷49	27÷29
$C_nH_{2\cdot n+1},\ n = \overline{7,10}.$	14	43	30

The second factor affecting the desired optimal amount of hydrogen fuel required for the life cycle of a single-cycle two-stroke gas engine is the kinetic properties of hydrogen. In a theoretical analysis of the working cycle of a gas engine running on hydrogen fuel, it is necessary to know the intensity of the turbulent combustion process (with the participation of an active radical OH) of homogeneous hydrogen-air mixtures for the coefficients α of excess air, where $\alpha \in [0.15, 3]$ (Nigmatulin, 1987a; 1987b). The intensity of radiation per unit of volume is proportional to the rate of deactivation of excited active radicals OH by a transition, the radiation of which is registered (Nigmatulin, 1987a; 1987b; Spalding, 1979):

$$I \sim \frac{dn^*}{d\tau}, \quad (1)$$

where n^* is the volume concentration of excited radicals.

Therefore, the total detonation radiation intensity is

$$I_\Sigma \sim \int\limits_{V_{\text{radiative}}} \frac{dn^*}{d\tau} dV, \quad (2)$$

where $V_{\text{radiative}}$ is the radiating volume, i.e. detonation volume.

In Table 2 the values of some kinetic parameters of hydrogen in the hydrogen-air mixture (i.e. directly in the engine/motor environment) are shown: the energy intensity of charge of the engine running on hydrogen; molecular mass; density; lower combustion temperature; laminar velocity of detonation propagation; ignition limits; stoichiometric hydrogen-air ratio; heat of combustion of a stoichiometric mixture; diffusion ability; boiling temperature; ignition energy; quenching zone thickness.

Table 2. Values of the main kinetic parameters of hydrogen in the hydrogen-air mixture directly in the motor environment.

Parameter	Unit of measurement	Value	
		H_2 (p=35 MPa)	**C_nH_{2n+1} (n=7,…,10)**
Molecular mass	kg/mol	2.015	117
Density	kJ/m^3	0.086	670
Heat of combustion	J/kg	120	44
The lowest combustion temperature	kg/m^3	10236	44000
	kJ/kg	120085	
Laminar velocity of detonation propagation	m/s	1.9÷2.7	0.37÷0.43
Ignition limits (fraction of volume)	%	4÷75	1.2÷6
Stoichiometric ratio of hydrogen and air (p=0.1 MPa, T=25 ºC)	m^3/kg	23.7	12.35
	m^3/m^3	2.38	50.06
	kg/kg	34.2	14.95
Heat of combustion of a stoichiometric mixture	kJ/kg	3029	3561
Molecular diffusion coefficient (p=0.1 MPa, T_0=19.85 ºC)	cm^2/s	0.63	0.085
Boiling temperature	ºC	–252.61	32÷215
Ignition energy	MJ	0.02	0.25
Quenching zone thickness	mm	0.6	2
Volume ratio of hydrogen and air	%	42	2

Since n^* is determined by the combustion reaction rate, then we can say that the volume concentration of excited radicals depends on the same parameters as the combustion rate, i.e.

$$n^* = n^*\left(\alpha, p, T_0\right), \quad (3)$$

where p is pressure, T_0 is the initial temperature, and these parameters are, generally speaking, functions.

If in the considered problem, following Williams (1985), we assume $p = p_{\text{const}} \equiv const$ and $T_0 = T_{\text{const}} \equiv const$, then the volume concentration of excited radicals

n^* will depend only on the excess air coefficients, i.e. $n^* = n^*(\alpha)$. Then, due to the fact that in laminar detonations of homogeneous mixtures the distribution of concentrations and temperatures over the volume is determined by the processes of molecular transfer, instead of formula (2) we can write the following formula:

$$I_\Sigma \sim \frac{dn^*_{\text{average}}(\alpha)}{d\tau_{\text{combustion}}} \cdot S_{\text{laminar}} \cdot \delta, \tag{4}$$

where $n^*_{\text{average}}(\alpha)$ is the volume average value of the concentration of active radicals $[OH]^*$; S_{laminar} is the laminar detonation area; δ is the laminar detonation thickness; $\tau_{\text{combustion}}$ is the characteristic burning time.

Denoting the normal detonation speed by $\vartheta_{\text{normal}}$, we get

$$\vartheta_{\text{normal}} = k \cdot \frac{\delta}{\tau_{\text{combustion}}}, \tag{5}$$

where k is some coefficient of proportionality, and denoting the volume flow rate (that is, the outgo rate) of the hydrogen-air mixture by $Q_{\text{mixture\_volume\_flow}}$ we get

$$Q_{\text{mixture\_volume\_flow}} = S_{\text{laminar}} \cdot \vartheta_{\text{normal}}, \tag{6}$$

so the formula (4), using (5) and (6) transforms into the following form (up to some proportionality coefficient):

$$I_\Sigma \sim n^*_{\text{average}}(\alpha) \cdot Q_{\text{mixture\_volume\_flow}}. \tag{7}$$

The resulting asymptotic formula (7) (here the word "asymptotic" is used in the sense of proportionality) allows us to state that the desired mass flow $G_{\text{mixture\_mass\_flow}}$ of hydrogen-air mixture is proportional to the total detonation radiation intensity, i.e.

$$G_{\text{mixture\_mass\_flow}} \sim I_\Sigma \sim n^*_{\text{average}}(\alpha) \cdot Q_{\text{mixture\_volume\_flow}}. \tag{8}$$

We emphasize once again that the asymptotic ratio was obtained under the conditions that both the pressure and the initial temperature are constants, i.e. $p = p_{\text{const}}$ and $T_0 = T_{\text{const}}$. Now, under the same conditions, we will try to find the mass flow $G_{\text{hydrogen\_mass\_flow}}$ of hydrogen in a hydrogen-air mixture with the coefficient $\alpha \in [0.15, 3]$ of excess air. When identifying a model gas with any real gas, it is sufficient (Shchetinkov, 1965; Saloukhin, 1965; 1966) to analyze only the macroparameters of the mixture included in the gas-dynamic equations, the total internal energy and molecular weight, moreover, the balance equations, i.e. equations approximating changes in these quantities should not contradict both the laws of conservation and the second law of thermodynamics and their formal analogues. Therefore, we can write

$$\left[(1-C_1) + \frac{k \cdot \Delta\alpha}{2} \cdot (1-C_2)\right] \cdot G_{\text{hydrogen\_mass\_flow}}(\alpha_{i+1}) +$$

$$+\left[C_2 \cdot (k \cdot \Delta\alpha)^2 + 2 \cdot C_1 - 2\right] \cdot G_{\text{hydrogen\_mass\_flow}}(\alpha_i) +$$

$$+\left[1 + (C_2 - C_1) \cdot \frac{k \cdot \Delta\alpha}{2}\right] \cdot G_{\text{hydrogen\_mass\_flow}}(\alpha_{i-1}) =$$

$$= C_3 \cdot \left\{\left(1 - \frac{k \cdot \Delta\alpha}{2}\right) \cdot G_{\text{mixture\_mass\_flow}}(\alpha_{i+1}) - 2 \cdot G_{\text{mixture\_mass\_flow}}(\alpha_i) +\right.$$

$$+\left(1+\frac{k\cdot\Delta\alpha}{2}\right)\cdot G_{\text{mixture\_mass\_flow}}\left(\alpha_{i-1}\right)\Bigg\},\ \ \forall i=\overline{1,N-1}, \tag{9}$$

where $N\in\mathbb{N},\ \alpha_0=0.15,\ \alpha_N=3,\ \Delta\alpha=\frac{\alpha_N-\alpha_0}{N},\ k$ is a proportionality factor; $C_i,\ i=\overline{1,3}$ are unknown constants, such that the equilibrium conditions must be satisfied

$$\left.\begin{aligned}&\sum_{i=1}^{3}C_i=1+k\cdot\alpha,\\&s_{\text{stoich.}}\cdot C_1-k\cdot\alpha\cdot C_2=1+k\cdot\alpha\cdot s_{\text{stoich.}}\cdot\left[C_2-C_3\right],\end{aligned}\right\} \tag{10}$$

where $s_{\text{stoichiometric}}$ is the stoichiometric coefficient.

From conditions (10) it is easy to determine the desired values $C_i,\ i=\overline{1,3}$:

$$C_1=\frac{1+s_{\text{stoich.}}\cdot k\cdot\alpha}{s_{\text{stoich.}}},\ \ C_2=1,\ \ C_3=-\frac{1}{s_{\text{stoich.}}}. \tag{11}$$

In equation (9), having properly passed to the limit $N\to\infty,$ we obtain

$$\begin{aligned}&G''_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)=C_1\left[G''_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)-G'_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)\right]+\\&+C_2\left[G'_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)-G_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)\right]+\\&+C_3\left[G''_{\text{mixture\_mass\_flow}}\left(\alpha\right)-G'_{\text{mixture\_mass\_flow}}\left(\alpha\right)\right],\end{aligned} \tag{9'}$$

Taking into account (11) in (9'), and after performing the necessary arithmetic operations, we obtain that the required mass flow rate $G_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)$ of hydrogen satisfies the second-order ordinary differential equation with constant coefficients

$$\left(1+s_{\text{stoich.}}\cdot k\cdot\alpha\right)\cdot G''_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)+2\cdot s_{\text{stoich.}}\cdot G'_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)-G''_{\text{mixture\_mass\_flow}}\left(\alpha\right)=0,$$

which has the following analytical solution (Pontryagin, 1974):

$$G_{\text{hydrogen\_mass\_flow}}\left(\alpha\right)=\frac{G_{\text{mixture\_mass\_flow}}\left(\alpha\right)}{1+s_{\text{stoich.}}\cdot k\cdot\alpha},\ \alpha\in\left[0.15,3\right]. \tag{12}$$

From formula (12) we obtain that in laminar diffusion detonations, where the combustion of a hydrogen-air mixture occurs near the surface of the stoichiometric composition and the structure of the reaction zone is also determined by molecular transfer processes, the following formula holds:

$$\frac{I_{\Sigma}}{G_{\text{hydrogen\_mass\_flow}}}\sim n^{*}_{\text{average}}\left(\alpha_{\text{const}}\right)\cdot\left(1+s_{\text{stoich.}}\cdot k\cdot\alpha_{\text{const}}\right)=const,\ \alpha_{\text{const}}\equiv const. \tag{13}$$

Therefore, we can say that there are dependencies similar to (13), when in a turbulent flow the turbulent motion affects only the curvature of the laminar detonation surface and the curvature does not yet affect its characteristics. However, both for laminar detonations in flows with large velocity gradients and for developed turbulent flows, the thickness of the excited zone decreases and may even disappear completely in laminar detonations subjected to tension in flows with strong gradients.

Calculation formulas (8), (12) and (13) can be obtained in a different way, using completely different considerations, which are based on the idea of using synthesis gas (Remark 2) that generates hydrogen. Dynamics of the amount of synthesis gas $z\left(t\right)$ (kg) generating hydrogen fuel can be described by the following balance equation (Nigmatulin, 1987a; 1987b; Williams, 1985):

$$a_i \cdot \frac{z_{i+1} - 2 \cdot z_i + z_{i-1}}{\tau^2} - \frac{(a_{i+1} - a_{i-1}) \cdot (z_{i+1} - z_{i-1})}{2 \cdot a_i \cdot \tau^2} + \frac{(a_{i+1} - a_{i-1})^2}{4 \cdot a_i \cdot \tau^2} \cdot z_i = \\ = \frac{a_{i+1} - 2 \cdot a_i + a_{i-1}}{\tau^2} \cdot z_i + w_i \cdot \frac{b_{i+1} - 2 \cdot b_i + b_{i-1}}{a_i \cdot \tau^2}, \; i \in \mathbb{N}, \qquad (13)$$

where

- $\tau = t_{j+1} - t_j$ is the length of the time interval during which the considered process is investigated;
- $a_j = a(t_j)$ is the specific consumption of synthesis gas per 1 kg of hydrogen fuel at moment of time t_j;
- $b_j = b(t_j)$ (kg/kW·h) is the specific consumption of hydrogen fuel per 1 kW·h of engine operation at moment of time t_j;
- w_j (kW·h) is engine's work at moment of time t_j.

Remark 2. The so-called synthesis gas can be obtained by steam reforming from natural gas, previously purified from iron carbonyls, sulphur compounds and oil particles, in accordance with the formula $CH_4 + H_2O \to CO + 3H_2 + 0.226\,MJ/mol$. *Another way to obtain synthesis gas is the process of partial oxidation of methane according to the formula* $CH_4 + 1/2\,O_2 \to CO + 2H_2 - 0.044\,MJ/mol$.

Assuming that the engine is working properly, i.e. assuming that $w_j \equiv const$ for $\forall j \in \mathbb{N}$, passing to limit $\tau \to 0$ in the balance equation (13) leads to the following ordinary differential equation of the 2nd order:

$$a^2(t) \cdot z''(t) - 2 \cdot a(t) \cdot a'(t) \cdot z'(t) + \left[(a'(t))^2 - a(t) \cdot a''(t) \right] \cdot z(t) = w \cdot b''(t), \qquad (14)$$

where

- $a(t)$ specific consumption of synthesis gas per 1 kg of hydrogen fuel,
- $b(t)$ (kg/kW·h) is the specific consumption of hydrogen fuel per 1 kW·h of engine's work;
- w (kW·h) is engine's work.

Let us assume that equation (14) is solved analytically and the function $z_{\text{synthesis\_gas}}(t)$ is its solution. Then the thermal energies of the synthesis gas $E_{\text{synthesis\_gas}}(t)$ (MJ) and of the used hydrogen fuel $E_{\text{hydrogen\_fuel}}(t)$ (MJ/kg) can be calculated using the following formulas, respectively:

$$E_{\text{synthesis\_gas}}(t) = T_{\text{synthesis\_gas}}^{\text{lovest\_heating\_power}} \cdot z_{\text{synthesis\_gas}}(t), \qquad (15)$$

$$E_{\text{hydrogen\_fuel}}(t) = T_{\text{hydrogen\_fuel}}^{\text{lovest\_heating\_power}} \cdot w \cdot b(t), \qquad (16)$$

where $T_{\text{synthesis\_gas}}^{\text{lovest\_heating\_power}}$ (MJ/kg) and $T_{\text{hydrogen\_fuel}}^{\text{lovest\_heating\_power}}$ (MJ/kg) are the lowest specific calorific value of synthesis gas and hydrogen fuel, respectively.

In the balance differential equation (14), we introduce the notation

$$y(t) = w \cdot b(t), \qquad (17)$$

the meaning of which is the amount of hydrogen fuel supplied to the engine for the process of use. Then the equation (14) splits into the following two related equations:

$$\begin{cases} z''(t)-a(t)\cdot y''(t)-2\cdot a'(t)\cdot y'(t)=a''(t)\cdot w\cdot b(t), \\ y'(t)=\dfrac{z'(t)-w\cdot a'(t)\cdot b(t)}{a(t)},\ a(t)\neq 0. \end{cases} \tag{18}$$

The system (18) together with the notation-formula (17) allows us to obtain the following compact calculation formula for calculating the amount of required synthesis gas $z(x)$ for hydrogen fuel engine operation:

$$z(t)=w\cdot a(t)\cdot b(t). \tag{19}$$

Similarly, using the laws of the material balance of the combustion process (Nigmatulin, 1987a; 1987b; Spalding, 1979; Williams, 1985; Shchetinkov, 1065), one can obtain the corresponding ordinary differential equations describing dynamics, for example,

– of electrical energy usage for the process of obtaining hydrogen fuel,

– of the amount of harmful substances emissions into the environment during the process of obtaining hydrogen fuel,

– of methane emissions during the process of obtaining hydrogen fuel,

– of thermal energy dissipation into the environment for the implementation of the process of obtaining hydrogen fuel, etc

The obtained differential equations can be solved by analytical methods (Pontryagin, 1974) or numerical methods, for example, by difference methods (Samarsky, 1977). It is obvious that a full-fledged study of the economic and environmental aspects of using hydrogen technology in transport power devices is impossible without accurate knowledge of the above (and many other) factors: no matter how beautiful or seemingly reasonable words are used to discuss these topics – the economic and environmental benefits/disadvantages of using hydrogen technology – without the involvement of mathematical apparatus, all these arguments will be just opinions (along with other opinions, sometimes even opposite ones), which do not have a solid, accurate, scientifically substantiated foundation. In this work, our goals are not to study the economic and environmental aspects of the use of hydrogen technology in transport power devices, however, we present some final calculation formulas that we obtained as a result of constructing and solving the corresponding mathematical models:

– Calculation formula for determining dynamics of usage of electrical energy $E_{\text{electric\_power}}$ (MJ) for the process of obtaining hydrogen fuel:

$$E_{\text{electric\_power}}(t)=\frac{1-\lambda_{\text{efficiency}}}{\lambda_{\text{efficiency}}}\cdot T_{\text{hydrogen\_fuel}}^{\text{lovest\_heating\_power}}\cdot U_{\text{relative\_energy\_consumption}}\cdot y(t),$$

where $\lambda_{\text{efficiency}}$ is the energy conversion efficiency of the hydrogen production process; $U_{\text{relative\_energy\_consumption}}$ is the relative energy consumption for the process of hydrogen production; $y(t)$ is determined by the formula (17).

– Calculation formula for determining dynamics of the quantity $H_{\text{exhaust\_emissions}}$ (kg) of emissions of harmful substances into the environment during the process of obtaining hydrogen fuel:

$$H_{\text{exhaust\_emissions}}(t)=\left(1-\lambda_{\text{efficiency}}\right)\cdot U_{\text{relative\_synthesis\_gas}}\cdot E_{\text{synthesis\_gas}}(t)\cdot\sum_i\theta_i(t)+$$
$$+\theta_{CH_4}(t)\cdot T_{\text{hydrogen\_fuel}}^{\text{lovest\_heating\_power}}\cdot y(t),$$

where $\theta_i(t)$ (kg/MJ) there is the specific emission of i-th harmful substance (or rather, emissions of exhaust gases, except for CH_4) in the process of obtaining hydrogen fuel; $\theta_{CH_4}(t)$ (kg/MJ) is the specific emission of methane due to the peculiarities of the technological process and possible leaks (hence, the term $\theta_{CH_4}(t)\cdot T_{\text{hydrogen\_fuel}}^{\text{lovest\_heating\_power}}\cdot y(t)$ determines the dynamics of the quantity (kg) of emission of CH_4 in the process of obtaining hydrogen fuel).

– Calculation formula for determining the dynamics of thermal energy dissipation into the environment for the implementation of the process of obtaining hydrogen fuel:

$$E_{\text{energy\_dissipation}}(t)=E_{\text{electric\_power}}(t)+\left(1-\lambda_{\text{efficiency}}\right)\cdot U_{\text{relative\_synthesis\_gas}}\cdot E_{\text{synthesis\_gas}}(t).$$

Below are some numerical results obtained by computer implementation of the calculation formula (12) for various values of the excess air coefficient. Since the number of parameter values α for small steps $\Delta\alpha$ of segment's $[0.15,3]$ variation is big (for example, for $\Delta\alpha=10^{-2}$ there are 286 values for the parameter α), then in the given numerical results, we, taking the step $\Delta\alpha=10^{-2}$, considered only options

– $\alpha\in[0.15,0.25]$ (This corresponds to the hydrogen-air mixture being a rich mixture),

– $\alpha\in[0.95,1.05]$ (This corresponds to the hydrogen-air mixture being a quasi-stoichiometric mixture: for the hydrogen-air mixture is a stoichiometric mixture),

– $\alpha\in[2.9,3]$ (This corresponds to the hydrogen-air mixture being a poor mixture).

It should also be noted that for each of these three options, we present the results only for the model values of 1 m^3, 3 m^3, 5 m^3 and 10 m^3 of the mass flow rate of the hydrogen-air mixture, i.e. $G_{\text{mixture\_mass\_flow}}=\{1;3;5;10\}$.

- For $G_{\text{mixture\_mass\_flow}}=1$ and $\alpha\in[0.15,0.25]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}0.76923\\0.75758\\0.74627\\0.73529\\0.72464\\0.71429\\0.70423\\0.69444\\0.68493\\0.67568\\0.66667\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}0.23077\\0.24242\\0.25373\\0.26471\\0.27536\\0.28571\\0.29577\\0.30556\\0.31507\\0.32432\\0.33333\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha) = \begin{pmatrix} 0.3 \\ 0.32 \\ 0.34 \\ 0.36 \\ 0.38 \\ 0.4 \\ 0.42 \\ 0.44 \\ 0.46 \\ 0.48 \\ 0.5 \end{pmatrix}. \tag{20}$$

- For $G_{\text{mixture\_mass\_flow}} = 1$ and $\alpha \in [0.95, 1.05]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha) = \begin{pmatrix} 0.34483 \\ 0.34247 \\ 0.34014 \\ 0.33784 \\ 0.33557 \\ 0.33333 \\ 0.33113 \\ 0.32895 \\ 0.3268 \\ 0.32468 \\ 0.32258 \end{pmatrix}, \quad G_{\text{air\_mass\_flow}}(\alpha) = \begin{pmatrix} 0.65517 \\ 0.65753 \\ 0.65986 \\ 0.66216 \\ 0.66443 \\ 0.66667 \\ 0.66887 \\ 0.67105 \\ 0.6732 \\ 0.67532 \\ 0.67742 \end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha) = \begin{pmatrix} 1.9 \\ 1.92 \\ 1.94 \\ 1.96 \\ 1.98 \\ 2 \\ 2.02 \\ 2.04 \\ 2.06 \\ 2.08 \\ 2.1 \end{pmatrix}. \tag{21}$$

- For $G_{\text{mixture\_mass\_flow}} = 1$ and $\alpha \in [2.9, 3]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}0.14706\\0.14663\\0.1462\\0.14577\\0.14535\\0.14493\\0.14451\\0.14409\\0.14368\\0.14327\\0.14286\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}0.85294\\0.85337\\0.8538\\0.85423\\0.85465\\0.85507\\0.85549\\0.85591\\0.85632\\0.85673\\0.85714\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}5.8\\5.82\\5.84\\5.86\\5.88\\5.9\\5.92\\5.94\\5.96\\5.98\\6\end{pmatrix}. \tag{22}$$

- For $G_{\text{mixture\_mass\_flow}}=3$ and $\alpha\in[0.15,0.25]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}2.30769\\2.27273\\2.23881\\2.20588\\2.17391\\2.14286\\2.11268\\2.08333\\2.05479\\2.02703\\2\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}0.69231\\0.72727\\0.76119\\0.79412\\0.82609\\0.85714\\0.88732\\0.91667\\0.94521\\0.97297\\1\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}0.3\\0.32\\0.34\\0.36\\0.38\\0.4\\0.42\\0.44\\0.46\\0.48\\0.5\end{pmatrix}. \quad (23)$$

- For $G_{\text{mixture\_mass\_flow}}=3$ and $\alpha\in[0.95,1.05]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}1.03448\\1.0274\\1.02041\\1.01351\\1.00671\\1\\0.99338\\0.98684\\0.98039\\0.97403\\0.96774\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}1.96552\\1.9726\\1.97959\\1.98649\\1.99329\\2\\2.00662\\2.01316\\2.01961\\2.02597\\2.03226\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}1.9\\1.92\\1.94\\1.96\\1.98\\2\\2.02\\2.04\\2.06\\2.08\\2.1\end{pmatrix}. \quad (24)$$

- For $G_{\text{mixture\_mass\_flow}}=3$ and $\alpha\in[2.9,3]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}0.44118\\0.43988\\0.4386\\0.43732\\0.43605\\0.43478\\0.43353\\0.43228\\0.43103\\0.4298\\0.42857\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}2.55882\\2.56012\\2.5614\\2.56268\\2.56395\\2.56522\\2.56647\\2.56772\\2.56897\\2.5702\\2.57143\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}5.8\\5.82\\5.84\\5.86\\5.88\\5.9\\5.92\\5.94\\5.96\\5.98\\6\end{pmatrix}. \tag{25}$$

- For $G_{\text{mixture\_mass\_flow}}=5$ and $\alpha\in[0.15,0.25]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}3.84615\\3.78788\\3.73134\\3.67647\\3.62319\\3.57143\\3.52113\\3.47222\\3.42466\\3.37838\\3.33333\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}1.15385\\1.21212\\1.26866\\1.32353\\1.37681\\1.42857\\1.47887\\1.52778\\1.57534\\1.62162\\1.66667\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}0.3\\0.32\\0.34\\0.36\\0.38\\0.4\\0.42\\0.44\\0.46\\0.48\\0.5\end{pmatrix}. \quad (26)$$

- For $G_{\text{mixture\_mass\_flow}}=5$ and $\alpha\in[0.95,1.05]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}1.72414\\1.71233\\1.70068\\1.68919\\1.67785\\1.66667\\1.65563\\1.64474\\1.63399\\1.62338\\1.6129\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}3.27586\\3.28767\\3.29932\\3.31081\\3.32215\\3.33333\\3.34437\\3.35526\\3.36601\\3.37662\\3.3871\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}1.9\\1.92\\1.94\\1.96\\1.98\\2\\2.02\\2.04\\2.06\\2.08\\2.1\end{pmatrix}. \quad (27)$$

- For $G_{\text{mixture\_mass\_flow}}=5$ and $\alpha\in[2.9,3]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}0.73529\\0.73314\\0.73099\\0.72886\\0.72674\\0.72464\\0.72254\\0.72046\\0.71839\\0.71633\\0.71429\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}4.26471\\4.26686\\4.26901\\4.27114\\4.27326\\4.27536\\4.27746\\4.27954\\4.28161\\4.28367\\4.28571\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}5.8\\5.82\\5.84\\5.86\\5.88\\5.9\\5.92\\5.94\\5.96\\5.98\\6\end{pmatrix}. \qquad (28)$$

- For $G_{\text{mixture\_mass\_flow}}=10$ and $\alpha\in[0.15,0.25]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}7.69231\\7.57576\\7.46269\\7.35294\\7.24638\\7.14286\\7.04225\\6.94444\\6.84932\\6.75676\\6.66667\end{pmatrix},\quad G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}2.30769\\2.42424\\2.53731\\2.64706\\2.75362\\2.85714\\2.95775\\3.05556\\3.15068\\3.24324\\3.33333\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}0.3\\0.32\\0.34\\0.36\\0.38\\0.4\\0.42\\0.44\\0.46\\0.48\\0.5\end{pmatrix}. \tag{29}$$

- For $G_{\text{mixture\_mass\_flow}}=10$ and $\alpha\in[0.95,1.05]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}3.44828\\3.42466\\3.40136\\3.37838\\3.3557\\3.33333\\3.31126\\3.28947\\3.26797\\3.24675\\3.22581\end{pmatrix},\ G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}6.55172\\6.57534\\6.59864\\6.62162\\6.6443\\6.66667\\6.68874\\6.71053\\6.73203\\6.75325\\6.77419\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}1.9\\1.92\\1.94\\1.96\\1.98\\2\\2.02\\2.04\\2.06\\2.08\\2.1\end{pmatrix}. \tag{30}$$

- For $G_{\text{mixture\_mass\_flow}}=10$ and $\alpha\in[2.9,3]$ we have:

$$G_{\text{hydrogen\_mass\_flow}}(\alpha)=\begin{pmatrix}1.47059\\1.46628\\1.46199\\1.45773\\1.45349\\1.44928\\1.44509\\1.44092\\1.43678\\1.43266\\1.42857\end{pmatrix},\ G_{\text{air\_mass\_flow}}(\alpha)=\begin{pmatrix}8.52941\\8.53372\\8.53801\\8.54227\\8.54651\\8.55072\\8.55491\\8.55908\\8.56322\\8.56734\\8.57143\end{pmatrix},$$

$$L_{\text{stoichiometric\_ratio}}(\alpha)=\begin{pmatrix}5.8\\5.82\\5.84\\5.86\\5.88\\5.9\\5.92\\5.94\\5.96\\5.98\\6\end{pmatrix}. \tag{31}$$

From the above shown numerical calculations, it can be seen that the stoichiometric volume (m^3/m^3) ratio/coefficient $L_{\text{stoichiometric\_volume\_ratio}}$ (as a function depending on the coefficient α of excess air) does not change at different values of the mass flow rate of the hydrogen-air mixture: for numerical results (20), (23), (26) and (29) $L_{\text{stoichiometric\_volume\_ratio}}$ is the same vector; for numerical results (21), (24), (27) and (30) $L_{\text{stoichiometric\_volume\_ratio}}$ is another vector; for numerical results (22), (25), (28) and (31) $L_{\text{stoichiometric\_volume\_ratio}}$ is again another vector. Figure 1 shows plots of dependencies of obtained stoichiometric volume (m^3/m^3) ratios/coefficients $L^{(20),(23),(26),(29)}_{\text{stoichiometric\_volume\_ratio}}$, $L^{(21),(24),(27),(30)}_{\text{stoichiometric\_volume\_ratio}}$, $L^{(22),(25),(28),(31)}_{\text{stoichiometric\_volume\_ratio}}$ of hydrogen-air mixture on the coefficient α of excess air. For ease of comparison, the same graph shows the plot (constant function) of the theoretical value (equal to 2.38); (Table 2) of the stoichiometric coefficient for the hydrogen-air mixture.

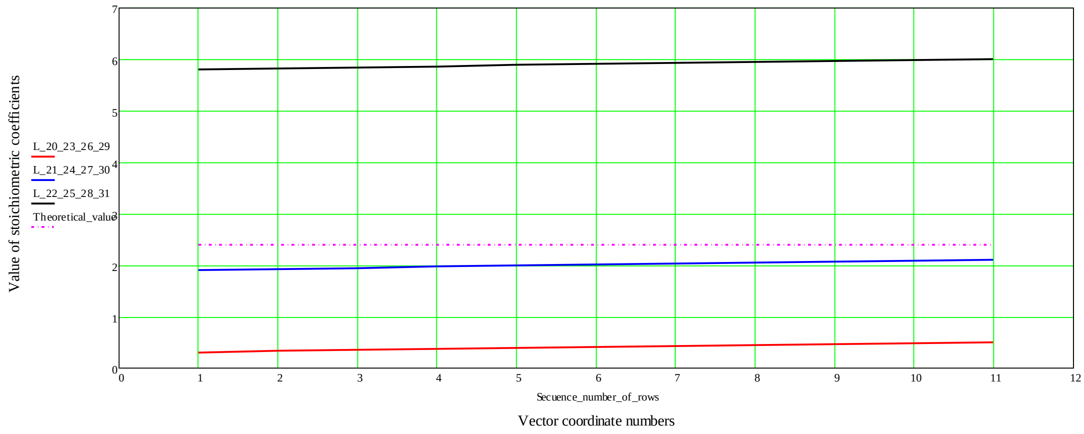


Figure 1. Comparison of the obtained stoichiometric coefficients with the theoretical stoichiometric coefficients of the hydrogen-air mixture

For a complete analysis (Kamm, 2002; United Nations: Inland Transport Committee, 2012; Heywood, 2018; Teh *et al.*, 2008) of the above numerical results, it is necessary to have sufficient information about the operating mode of a particular hydrogen power device, about its operating conditions, about the method of supplying hydrogen and oxidizing air to the main combustion chamber, about the type of the combustion chamber itself, about the type/method of ignition, about the type of propulsion mechanism of the power device (piston, turbojet, ramjet, etc.), about the detonation resistance of the hydrogen fuel used, which can be obtained in various ways (Remark 1). For example, for aircraft power devices operating on hydrogen technology using the so-called light hydrogen-air mixture, the operational value of the excess air coefficient, depending on the operating mode and degree of loading, varies within 0.6÷1, and for piston aircraft power devices – within 1.5÷2.5. Therefore, from the graphs shown in Figure 1, it does not follow at all that $\alpha \in [0.95, 1.05]$ (corresponds to the blue graph) is the optimal interval for the considered problem due to the fact that this graph is closer to the theoretical value of the stoichiometric coefficient of the hydrogen-air mixture. Despite the fact that hydrogen power devices operating on a hydrogen-air mixture with the principle of internal combustion need a much larger amount of air (as can be seen from Table 1, the theoretically optimal air-fuel ratio for gasoline power plants is 14.3:1 and for hydrogen power devices – 38:1) than similar principle of operation of power devices using gasoline fuel, excessive air consumption non-linearly reduces the power of the power mechanism: due to the fact that hydrogen is a more energy-intensive fuel (Table 2), a power device operating on the principle of internal combustion needs approximately 1.5 times the amount of hydrogen. And this means that in the main combustion chamber the temperature will rise approximately the same number of times, which leads to the breaking of the triple bonds of nitrogen in the air and as a result, nitrogen oxide N_2O, which is one of the most harmful environmental pollutants (this substance, which is sometimes called "laughing gas", is a narcotic drug used in surgery, operative gynaecology, birth pain relief and other cases), begins to form. It turns out to be in some way a vicious circle: an increase of air in the hydrogen-air mixture generates power losses in the power mechanism of a hydrogen power device and its decrease causes environmental pollution.

In addition to the above circumstances, there is one more circumstance – an increase of the risk of detonation and explosion of the hydrogen-air mixture.

Therefore, the main task of the safe and economically-environmentally beneficial use of hydrogen power devices is to improve the principle of their operation, bearing in mind that this principle is based on a chemical reaction occurring in a hydrogen fuel cell. However, the study of this problem is not included in the purpose of our present study: the study is carried out on a given engine using hydrogen technology. Now let us briefly dwell on both the principle of operation of a hydrogen power device and on the basis of this principle. For this, we consider a model hydrogen power device operating on the principle of internal combustion and having a spark ignition system and a piston power mechanism. Such a power device is a set of interacting elements, which include (1) a power mechanism containing a block of cylinder-piston groups and a crankshaft; (2) fuel equipment and combustion chamber; (3) consumer – a mechanism for the conversion and consumption of energy (in what follows we will call these elements engine parts). Interactions of engine parts generate input and output coordinates : for example, for the power mechanism the input coordinates are cycling hydrogen fuel supply, cycling air supply and load, but the output coordinates are the angular velocity of the crankshaft and the gas supply to the exhaust manifold, for fuel equipment the input coordinates are the position of the rack control and the angular velocity of the camshaft and the output coordinates are the cyclic supply of hydrogen fuel, etc. (Heywood, 2018; The *et al.*, 2008).

The mode of operation of a hydrogen power device operating on the principle of internal combustion and having a spark ignition system and a piston power mechanism, is its state (i.e., the cumulative state of interconnected and mutually influencing engine parts) in the process of operation. This state, i.e. operating mode of the power device, is characterized by the following main parameters (Heywood, 2018; The *et al.*, 2008; Krutov, 1989):

1) Effective (braking) power (kW),
2) Torque on the power take-off shaft (N·m),
3) Angular speed of the crankshaft (min^{-1}),
4) Boost pressure (MPa),
5) Hydrogen fuel consumption (L/h; kg/h; L/km; m^3/h; m^3/km),
6) Specific consumption of hydrogen fuel (g/kW·h),
7) Effective specific consumption of the hydrogen-air mixture (kg/kW·h)
8) Excess air coefficient (dimensionless),
9) Stoichiometric coefficient of hydrogen-air mixture (dimensionless),
10) Effective (braking) energy conversion efficiency (dimensionless),
11) Indicator energy conversion efficiency (dimensionless),
12) Mechanical energy conversion efficiency(dimensionless),
13) Cylinder diameter (cm),
14) Frequency of rotation (Hz; min^{-1}),
15) The degree of forcing the engine (kW/cm^2),
16) Atmospheric pressure (MPa),
17) Pressure at the end of compression (MPa),
18) Maximum combustion pressure (MPa),
19) Average effective pressure (MPa),
20) Average indicator pressure (MPa),
21) Piston stroke (cm; m),

22) Ambient temperature (ºC),
23) Air temperature after compressor (ºC),
24) Exhaust gas temperature (ºC),
25) Space compression volume (L; dm^3; m^3),
26) Working volume of cylinder (L; dm^3; m^3),
27) Total cylinder volume (L; dm^3; m^3),
28) Average piston speed (m/s),
29) Nominal compression rate (dimensionless),
30) Air humidity (%).

Generally speaking, if we cluster the above 3 engine parts according to the input and output coordinates of the units, then 8 blocks of parts can be conditionally identified (in what follows, for brevity, these blocks of parts will be called parts). Each of the above 30 main parameters of the hydrogen power device operation mode affects the operating state of both "own cluster"/"own part" and "foreign cluster"/"foreign part" and each parameter has its own/individual weight of influence. To quantify the efficiency of a power device operation mode, finding these influence weights of parameter is an important task and there are currently various approaches to solving this problem. However, all these methods have a common scheme, which can be represented as a functional

$$E\left[y;t\right] \overset{def}{\equiv} F\left(y_1,...,y_m;C\right)(t), \tag{32}$$

where $m \in \mathbb{N}$ is the number of elements of the power device; $y_i = f_i\left(x_1,...,x_n;t\right)$ there is a technical indicator of i-th $\left(i=\overline{1,m}\right)$ element of the power device, called i-th partial indicator of quality; $n \in \mathbb{N}$ is the number of controlled/calculated (mainly structural and constructive) parameters of the power device; $\left\{x_j\right\}_{j=1}^{j=n}$ are controlled/calculated parameters of the power device; t is, obviously, time; C stands for the total cost of operating the power device.

As can be seen from (32), the difference between the methods for finding the efficiency $E\left[y;t\right]$ is due to specific types of functions $f_i\left(x_1,...,x_n;t\right)$ $\left(i=\overline{1,m}\right)$ and F. Namely, constructing functions $f_i\left(x_1,...,x_n;t\right)$ $\left(i=\overline{1,m}\right)$ and F in various ways, we obtain various formulas for calculating E, moreover, if the constructed functions $f_i\left(x_1,...,x_n;t\right)$ $\left(i=\overline{1,m}\right)$ and F have an analytical form, then the calculation $E\left[y;t\right]$ does not present any difficulty, because the controlled/calculated parameters of the considered power device are known for each variant/state of this system; if the form of these functions is unknown, then in order to calculate $E\left[y;t\right]$ one mainly uses statistical methods and sometimes it all is limited to estimating $E\left[y;t\right]$ according to only one most important private indicator of quality $y_{imp.}$ $\left(imp \in \left\{1;...;m\right\}\right)$ and restrictions are imposed on other private quality indicators so that they do not go beyond certain limits:

$$\begin{cases} E\left[y;t\right] = y_{imp.}, \\ \underline{y_i} \le y_i \le \overline{y_i} \quad \left(i=\overline{1,m}/\left\{imp\right\}\right), \end{cases} \tag{33}$$

where $\underline{y_i} \overset{def}{\equiv} \inf_{x;t} f_i\left(x_1,\ldots,x_n;t\right)$ and $\overline{y_i} \overset{def}{\equiv} \sup_{x;t} f_i\left(x_1,\ldots,x_n;t\right)$ are respectively the lower and upper limits of i-th $\left(i=\overline{1,m}\right)$ private quality indicator (it should be noted that depending on the private quality indicator, one of the limits $\underline{y_i}$, $\overline{y_i}$ might be unbounded).

Evaluation of the efficiency of a power device according to the rule (33) has the same disadvantage as the solution of the corresponding optimization problem (i.e. the same problem (33) with the criterion $E\left[y;t\right]=y_{imp.}\to\sup$) or the problem of choosing variant of power device for the implementation of practical optimization. It is ambiguous, because the choice of the criterion itself is ambiguous: it is possible to obtain many options for the operation mode of a power device with the same or almost the same main partial quality indicator $y_{imp.}$ with significantly different other private indicators that satisfy the restrictions; therefore, it is impossible to determine with certainty which variant of the power device operation mode will be closer to the optimal variant.

One of the widely used and well-studied methods for estimating the efficiency of a power device is the weighted coefficient method, in which the function F is represented as a linear combination of functions y_i $\left(i=\overline{1,m}\right)$:

$$\left.\begin{aligned} &E\left[y;t\right]\overset{def}{\equiv}\sum_{i=1}^{m}\lambda_i\cdot y_i,\ \lambda_i\in\Lambda\overset{def}{\equiv}\left\{\lambda_i\in\mathbb{R}_+^1:\ \sum_{i=1}^{m}\lambda_i=1\right\}\ \forall i=\overline{1,m},\\ &\underline{y_i}=\inf_{x;t} f_i\left(x_1,\ldots,x_n;t\right)\le y_i\le\overline{y_i}=\sup_{x;t} f_i\left(x_1,\ldots,x_n;t\right)\ \forall i=\overline{1,m}.\end{aligned}\right\} \tag{34}$$

In (34) the weighted coefficient $\lambda_i\in\Lambda$ is called importance/significance coefficient of i-th $\left(i=\overline{1,m}\right)$ partial indicator of quality and generally speaking, is the desired number; the set Λ is called the importance or preference set.

It should be noted here that in (33) sometimes instead of $2\cdot n$ inequalities stricter restrictions are used $y_i\in\left[y_{\min},y_{\max}\right]$ $\left(i=\overline{1,m}\right)$, where $y_{\min}\overset{def}{\equiv}\underline{\min}_{i=1,m}\inf_{x;t} f_i\left(x_1,\ldots,x_n;t\right)$ and $y_{\max}\overset{def}{\equiv}\underline{\max}_{i=1,m}\sup_{x;t} f_i\left(x_1,\ldots,x_n;t\right)$.

In formula (34), partial quality indicators $\left\{y_i\right\}_{i=1}^{i=m}$ are chosen in such a way that when the considered power device approaches the practical optimal system, they all decrease or increase: then for the practical optimal power device, the overall efficiency indicator $E\left[y;t\right]$ will have a minimum or maximum value, respectively.

We emphasize once again that both determining the coefficients of importance of particular indicators of quality of a power device and the problem of finding/evaluating these particular indicators of the system themselves are independent problems. To this date, quite a lot of analytical, analytical-numerical, logical, statistical, network-graphical, expert, etc. methods for solving these independent problems have been developed and they are successfully applied. Comprehensive information about the main and most common of these methods, as well as a fairly complete overview of them with reasonable indications of advantages, disadvantages and scope can be found in Avduyevsky et al. (1988); Ushakov et al. (1985); Chumakov & Serebryaniy (1980); Greshilov (2006); Muschick & Müller (1986); Berezovsky & Gnedin (1984); Kostogrizov & Nistratov (2004); Totsenko (2002); Lotov & Pospelova (2008); Andreychikov & Andreychikova

(2013); Rykov & Itkin (2009). Therefore, here we will not deal with the problem of finding controlled/calculated structural and design parameters $\{x_j\}_{j=1}^{j=n}$ power device, nor the problem of evaluating partial quality indicators $\{y_i\}_{j=1}^{i=m}$ of elements of the power device: we will assume that all their values are known to us a posteriori or taken from the relevant literature (Heywood, 2018; Teh *et al.*, 2008; Krutov, 1989).

In this work, when conducting computer experiments to find the weights of influence of the above parameters 1)-30), we used the approach described in Guseynov et al. (2015): this approach guarantees the stability of the obtained numerical results (i.e., inevitable small errors in the values of the initial data, which are the results of measurements, do not distort the essence of the obtained numerical results). Computer implementation of the corresponding calculation formulas from Guseynov et al. (2015) with the initial data from Heywood (2018); Teh et al. (2008); Krutov (1989) gives us the following results (see [A]-[E]):

[A] Weights of all 30 parameters within the operating mode of a hydrogen power device:

$$\text{Weight of parameters} = \begin{pmatrix} 0.74224 \\ 0.65094 \\ 1.46704 \\ 0.48959 \\ 0.49260 \\ 0.77575 \\ 0.74983 \\ 0.51518 \\ 0.56396 \\ 1.03425 \\ 1.16053 \\ 0.65963 \\ 0.61162 \\ 0.53800 \\ 0.51067 \\ 0.43907 \\ 0.65537 \\ 0.81312 \\ 0.43011 \\ 0.57075 \\ 0.50998 \\ 0.47132 \\ 0.51227 \\ 0.66215 \\ 0.64066 \\ 0.58821 \\ 0.43681 \\ 0.51725 \\ 0.45977 \\ 0.44394 \end{pmatrix}, \quad \text{Weight (standardized) of parameters} = \begin{pmatrix} 0.3709 \\ 0.3299 \\ 1 \\ 0.273 \\ 0.3227 \\ 0.6098 \\ 0.5835 \\ 0.3456 \\ 0.3951 \\ 0.8719 \\ 1 \\ 0.5786 \\ 0.5192 \\ 0.4281 \\ 0.3942 \\ 0.3056 \\ 0.5734 \\ 0.7686 \\ 0.2945 \\ 0.4686 \\ 0.3934 \\ 0.3455 \\ 0.3962 \\ 0.5818 \\ 0.5552 \\ 0.4902 \\ 0.3028 \\ 0.4024 \\ 0.3312 \\ 0.3116 \end{pmatrix}.$$

Remark to [A]. The first column of the given numerical calculations shows the original weight values (i.e., the original, unprocessed weight values), and the second column shows the standardized in the segment $[0.1;1]$ (i.e., using minimax standardization with a shift) weight values of all 30 parameters within mode of operation of a hydrogen power device.

[B] Weights of all 8 parts within the operating mode of a hydrogen power device for the total of all 30 parameters:

$$\text{Overall impact of details} = \begin{pmatrix} 7.96331932937623 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 8.61426541111905 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 14.2921664308140 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 11.4743886952239 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 9.26478246614387 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\ 6.77958958370586 \leftarrow \text{Detail\_6\_Engine\_part\_3} \\ 10.3440580468547 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 11.2674300367625 \leftarrow \text{Detail\_8\_Engine\_part\_3} \end{pmatrix}.$$

Remark to [B]. The above numerical calculations show that, according to the cumulative influence of parameters 1)-30), part #3, which is an element of engine part #1, has the largest weight among all 8 parts of the hydrogen power device and part #6, which is an element of engine part #3, has the smallest weight among all 8 parts of a hydrogen power device.

[C] Weights of all 3 engine parts within the operating mode of a hydrogen power device for the total of all 30 parameters:

$$\text{Overall impact of engine parts} = \begin{pmatrix} 9983.07377612381 \leftarrow \text{Engine\_part\_3} \\ 9982.80211546073 \leftarrow \text{Engine\_part\_2} \\ 9982.33295056359 \leftarrow \text{Engine\_part\_1} \end{pmatrix}.$$

Remark to [C]. The given numerical calculations show that according to the cumulative influence of parameters 1)-30) all 3 engine parts have almost the same influence on the operation mode of the hydrogen power device.

[D] The weight of influence of each of the 30 parameters for each of the 3 engine parts of the hydrogen power device:

$$\begin{pmatrix}
\text{Weight of the parameter P1 for engine part 1 is equal to 0.74228; for engine part 2 is equal to 0.74230; for engine part 3 is equal to 0.74213} \\
\text{Weight of the parameter P2 for engine part 1 is equal to 0.65094; for engine part 2 is equal to 0.65089; for engine part 3 is equal to 0.65098} \\
\text{Weight of the parameter P3 for engine part 1 is equal to 1.46703; for engine part 2 is equal to 1.46704; for engine part 3 is equal to 1.46704} \\
\text{Weight of the parameter P4 for engine part 1 is equal to 0.48957; for engine part 2 is equal to 0.48962; for engine part 3 is equal to 0.48958} \\
\text{Weight of the parameter P5 for engine part 1 is equal to 0.49256; for engine part 2 is equal to 0.49261; for engine part 3 is equal to 0.4926} \\
\text{Weight of the parameter P6 for engine part 1 is equal to 0.77571; for engine part 2 is equal to 0.77571; for engine part 3 is equal to 0.77583} \\
\text{Weight of the parameter P7 for engine part 1 is equal to 0.74980; for engine part 2 is equal to 0.74979; for engine part 3 is equal to 0.74990} \\
\text{Weight of the parameter P8 for engine part 1 is equal to 0.51508; for engine part 2 is equal to 0.51527; for engine part 3 is equal to 0.51517} \\
\text{Weight of the parameter P9 for engine part 1 is equal to 0.56387; for engine part 2 is equal to 0.56406; for engine part 3 is equal to 0.56396} \\
\text{Weight of the parameter P10 for engine part 1 is equal to 1.03416; for engine part 2 is equal to 1.03435; for engine part 3 is equal to 1.03423} \\
\text{Weight of the parameter P11 for engine part 1 is equal to 1.16052; for engine part 2 is equal to 1.16036; for engine part 3 is equal to 1.16070} \\
\text{Weight of the parameter P12 for engine part 1 is equal to 0.65948; for engine part 2 is equal to 0.65977; for engine part 3 is equal to 0.65964} \\
\text{Weight of the parameter P13 for engine part 1 is equal to 0.61156; for engine part 2 is equal to 0.61164; for engine part 3 is equal to 0.61166} \\
\text{Weight of the parameter P14 for engine part 1 is equal to 0.53761; for engine part 2 is equal to 0.53810; for engine part 3 is equal to 0.53830} \\
\text{Weight of the parameter P15 for engine part 1 is equal to 0.51063; for engine part 2 is equal to 0.51065; for engine part 3 is equal to 0.51073} \\
\text{Weight of the parameter P16 for engine part 1 is equal to 0.43905; for engine part 2 is equal to 0.43904; for engine part 3 is equal to 0.43912} \\
\text{Weight of the parameter P17 for engine part 1 is equal to 0.65586; for engine part 2 is equal to 0.65541; for engine part 3 is equal to 0.65483} \\
\text{Weight of the parameter P18 for engine part 1 is equal to 0.81314; for engine part 2 is equal to 0.81313; for engine part 3 is equal to 0.81308} \\
\text{Weight of the parameter P19 for engine part 1 is equal to 0.43011; for engine part 2 is equal to 0.43011; for engine part 3 is equal to 0.43011} \\
\text{Weight of the parameter P20 for engine part 1 is equal to 0.57079; for engine part 2 is equal to 0.57076; for engine part 3 is equal to 0.57069} \\
\text{Weight of the parameter P21 for engine part 1 is equal to 0.51003; for engine part 2 is equal to 0.50996; for engine part 3 is equal to 0.50995} \\
\text{Weight of the parameter P22 for engine part 1 is equal to 0.47137; for engine part 2 is equal to 0.47135; for engine part 3 is equal to 0.47123} \\
\text{Weight of the parameter P23 for engine part 1 is equal to 0.51223; for engine part 2 is equal to 0.51232; for engine part 3 is equal to 0.51226} \\
\text{Weight of the parameter P24 for engine part 1 is equal to 0.66212; for engine part 2 is equal to 0.66210; for engine part 3 is equal to 0.66223} \\
\text{Weight of the parameter P25 for engine part 1 is equal to 0.64059; for engine part 2 is equal to 0.64062; for engine part 3 is equal to 0.64078} \\
\text{Weight of the parameter P26 for engine part 1 is equal to 0.58829; for engine part 2 is equal to 0.58819; for engine part 3 is equal to 0.58815} \\
\text{Weight of the parameter P27 for engine part 1 is equal to 0.43685; for engine part 2 is equal to 0.43663; for engine part 3 is equal to 0.43696} \\
\text{Weight of the parameter P28 for engine part 1 is equal to 0.51697; for engine part 2 is equal to 0.51735; for engine part 3 is equal to 0.51741} \\
\text{Weight of the parameter P29 for engine part 1 is equal to 0.45977; for engine part 2 is equal to 0.45977; for engine part 3 is equal to 0.45977} \\
\text{Weight of the parameter P30 for engine part 1 is equal to 0.44397; for engine part 2 is equal to 0.44384; for engine part 3 is equal to 0.44401}
\end{pmatrix}.$$

[E] Influence of details and engine parts of a hydrogen power device for each of the 30 parameters:

$$\left.\begin{matrix}
\text{Impact of details for parameter \#1} = \begin{pmatrix}
2.44651490547001 \leftarrow \text{Detail_5_Engine_part_2} \\
1.27237838190739 \leftarrow \text{Detail_3_Engine_part_1} \\
1.19402026239836 \leftarrow \text{Detail_4_Engine_part_1} \\
0.870369349602023 \leftarrow \text{Detail_1_Engine_part_1} \\
0.857103938596308 \leftarrow \text{Detail_6_Engine_part_3} \\
0.639371272067768 \leftarrow \text{Detail_7_Engine_part_3} \\
0.49667675208897 \leftarrow \text{Detail_2_Engine_part_1} \\
0.223565137869177 \leftarrow \text{Detail_8_Engine_part_3}
\end{pmatrix} \\
\text{Impact of engine parts for parameter \#1} = \begin{pmatrix}
997.796366868769 \leftarrow \text{Engine_part_2} \\
997.796366868713 \leftarrow \text{Engine_part_3} \\
997.796366868528 \leftarrow \text{Engine_part_1}
\end{pmatrix}
\end{matrix}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#2} = \begin{pmatrix} 2.13688708328734 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.49094006600146 \leftarrow \text{Detail_2_Engine_part_1} \\ 1.23539958550398 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.19938578264593 \leftarrow \text{Detail_1_Engine_part_1} \\ 1.12316796148102 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.382118803512903 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.23345209431143 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.19864862325594 \leftarrow \text{Detail_5_Engine_part_2} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#2} = \begin{pmatrix} 998.050782094069 \leftarrow \text{Engine_part_1} \\ 998.048334515343 \leftarrow \text{Engine_part_3} \\ 998.045845259328 \leftarrow \text{Engine_part_2} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#3} = \begin{pmatrix} 4.84121603142619 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.482589975823937 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.455557159955826 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.447189507424343 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.446263625453882 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.444146846418612 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.442883243830862 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.44015360966635 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#3} = \begin{pmatrix} 996.908870472129 \leftarrow \text{Engine_part_3} \\ 996.908869755302 \leftarrow \text{Engine_part_2} \\ 996.908847455149 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#4} = \begin{pmatrix} 1.60785414560188 \leftarrow \text{Detail_1_Engine_part_1} \\ 1.57423375016758 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.37531809016656 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.963415256685155 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.902132496881588 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.762448984579003 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.665591715683888 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.14900556023435 \leftarrow \text{Detail_8_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#4} = \begin{pmatrix} 998.732021118417 \leftarrow \text{Engine_part_3} \\ 998.728194684434 \leftarrow \text{Engine_part_2} \\ 998.709897719003 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left\{\begin{array}{l}\text{Impact of details for parameter \#5} = \begin{pmatrix} 1.61033515810027 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\ 1.38885294896384 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 1.30306760222054 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.947960110292541 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 0.916870936746021 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 0.900831334129054 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 0.760641880208284 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 0.171440029339444 \leftarrow \text{Detail\_6\_Engine\_part\_3} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#5} = \begin{pmatrix} 998.743750308714 \leftarrow \text{Engine\_part\_3} \\ 998.729336743932 \leftarrow \text{Engine\_part\_2} \\ 998.647153689831 \leftarrow \text{Engine\_part\_1} \end{pmatrix}\end{array}\right\};$$

$$\left\{\begin{array}{l}\text{Impact of details for parameter \#6} = \begin{pmatrix} 2.3609253920367 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 1.59872925002388 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 0.963144128757976 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 0.897256589578543 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.88566695691594 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 0.589049914934705 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 0.458187065739823 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\ 0.247040702012436 \leftarrow \text{Detail\_6\_Engine\_part\_3} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#6} = \begin{pmatrix} 997.816376292541 \leftarrow \text{Engine\_part\_3} \\ 997.670682232741 \leftarrow \text{Engine\_part\_1} \\ 997.669225551713 \leftarrow \text{Engine\_part\_2} \end{pmatrix}\end{array}\right\};$$

$$\left\{\begin{array}{l}\text{Impact of details for parameter \#7} = \begin{pmatrix} 2.28785263811045 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 1.57851858994235 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 0.981736870724105 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 0.919716577110604 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 0.899030487740216 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.612194248700995 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 0.487681105756184 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\ 0.233269481915089 \leftarrow \text{Detail\_6\_Engine\_part\_3} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#7} = \begin{pmatrix} 997.869817223524 \leftarrow \text{Engine\_part\_3} \\ 997.736009739859 \leftarrow \text{Engine\_part\_1} \\ 997.728712550857 \leftarrow \text{Engine\_part\_2} \end{pmatrix}\end{array}\right\};$$

$$\left.\begin{array}{l}\text{Impact of details for parameter \#8} = \begin{pmatrix} 1.53615237103375 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 1.4826932286353 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 1.11963669969554 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 1.10462229051409 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 1.06008877167221 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.735219270287856 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 0.494225461838854 \leftarrow \text{Detail\_6\_Engine\_part\_3} \\ 0.46736190632241 \leftarrow \text{Detail\_5\_Engine\_part\_2} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#8} = \begin{pmatrix} 998.777584745763 \leftarrow \text{Engine\_part\_2} \\ 998.581144067797 \leftarrow \text{Engine\_part\_3} \\ 998.405550847458 \leftarrow \text{Engine\_part\_1} \end{pmatrix}\end{array}\right\};$$

$$\left.\begin{array}{l}\text{Impact of details for parameter \#9} = \begin{pmatrix} 1.6059445544351 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 1.26633924463403 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 1.12932300196056 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 1.08960409094693 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 1.08265691379459 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.795707650239298 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 0.541529601398742 \leftarrow \text{Detail\_6\_Engine\_part\_3} \\ 0.488894942590755 \leftarrow \text{Detail\_5\_Engine\_part\_2} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#9} = \begin{pmatrix} 998.52764084507 \leftarrow \text{Engine\_part\_2} \\ 998.361267605634 \leftarrow \text{Engine\_part\_3} \\ 998.203697183099 \leftarrow \text{Engine\_part\_1} \end{pmatrix}\end{array}\right\};$$

$$\left.\begin{array}{l}\text{Impact of details for parameter \#10} = \begin{pmatrix} 2.83728490013764 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\ 1.21124620752628 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\ 0.950021490731241 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\ 0.928693271233395 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\ 0.704774927930371 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\ 0.49153464408909 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\ 0.491532040142697 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\ 0.384912518209293 \leftarrow \text{Detail\_6\_Engine\_part\_3} \end{pmatrix} \\ \text{Impact of engine parts for parameter \#10} = \begin{pmatrix} 997.389690721649 \leftarrow \text{Engine\_part\_2} \\ 997.273711340206 \leftarrow \text{Engine\_part\_3} \\ 997.20412371134 \leftarrow \text{Engine\_part\_1} \end{pmatrix}\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#11} = \begin{pmatrix} 3.21083831684335 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.35394542964881 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.04447431676043 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.673133809993492 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.673133809993492 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.34815810558681 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.34815810558681 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.34815810558681 \leftarrow \text{Detail_1_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#11} = \begin{pmatrix} 997.3 \leftarrow \text{Engine_part_3} \\ 997.14 \leftarrow \text{Engine_part_1} \\ 997.006666666667 \leftarrow \text{Engine_part_2} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#12} = \begin{pmatrix} 1.82586098388045 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.42843757499387 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.03906738586599 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.01486047837426 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.99267201848512 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.9200521799051 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.450041074081758 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.329008304413454 \leftarrow \text{Detail_3_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#12} = \begin{pmatrix} 998.233333333333 \leftarrow \text{Engine_part_2} \\ 998.034556574924 \leftarrow \text{Engine_part_3} \\ 997.796024464832 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#13} = \begin{pmatrix} 1.94496361287362 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.58655672297411 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.40652912344383 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.959099556631231 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.957491562078369 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.557991581015167 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.364336975201575 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.2230308657821 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#13} = \begin{pmatrix} 998.251230833856 \leftarrow \text{Engine_part_3} \\ 998.211573630109 \leftarrow \text{Engine_part_2} \\ 998.077174083943 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\}$$

$$
\left.\begin{array}{l}
\text{Impact of details for parameter \#14} = \begin{pmatrix}
1.42846603645953 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\
1.30686665793674 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\
1.15092896434496 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\
1.12467098695977 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\
0.88177031170207 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\
0.820909114541555 \leftarrow \text{Detail\_6\_Engine\_part\_3} \\
0.751746957450984 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\
0.534640970604399 \leftarrow \text{Detail\_2\_Engine\_part\_1}
\end{pmatrix} \\
\text{Impact of engine parts for parameter \#14} = \begin{pmatrix}
998.930779215515 \leftarrow \text{Engine\_part\_3} \\
998.556392985509 \leftarrow \text{Engine\_part\_2} \\
997.641935483871 \leftarrow \text{Engine\_part\_1}
\end{pmatrix}
\end{array}\right\};
$$

$$
\left.\begin{array}{l}
\text{Impact of details for parameter \#15} = \begin{pmatrix}
1.62936233048164 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\
1.62138344294466 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\
1.39796508778616 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\
1.12667317331406 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\
0.879311554158749 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\
0.695793254885919 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\
0.484340936934269 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\
0.16517021949454 \leftarrow \text{Detail\_6\_Engine\_part\_3}
\end{pmatrix} \\
\text{Impact of engine parts for parameter \#15} = \begin{pmatrix}
998.73203125 \leftarrow \text{Engine\_part\_3} \\
998.56796875 \leftarrow \text{Engine\_part\_2} \\
998.5328125 \leftarrow \text{Engine\_part\_1}
\end{pmatrix}
\end{array}\right\};
$$

$$
\left.\begin{array}{l}
\text{Impact of details for parameter \#16} = \begin{pmatrix}
1.40453644167845 \leftarrow \text{Detail\_3\_Engine\_part\_1} \\
1.38973417412905 \leftarrow \text{Detail\_5\_Engine\_part\_2} \\
1.19239820729546 \leftarrow \text{Detail\_8\_Engine\_part\_3} \\
1.1874733090705 \leftarrow \text{Detail\_4\_Engine\_part\_1} \\
0.995067075264591 \leftarrow \text{Detail\_1\_Engine\_part\_1} \\
0.879131371687852 \leftarrow \text{Detail\_7\_Engine\_part\_3} \\
0.652191611977971 \leftarrow \text{Detail\_2\_Engine\_part\_1} \\
0.299467808896123 \leftarrow \text{Detail\_6\_Engine\_part\_3}
\end{pmatrix} \\
\text{Impact of engine parts for parameter \#16} = \begin{pmatrix}
999.130366685301 \leftarrow \text{Engine\_part\_3} \\
998.964606741573 \leftarrow \text{Engine\_part\_1} \\
998.933707865169 \leftarrow \text{Engine\_part\_2}
\end{pmatrix}
\end{array}\right\};
$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#17} = \begin{pmatrix} 1.81395338399604 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.06580557787638 \leftarrow \text{Detail_5_Engine_part_2} \\ 1.02326208376977 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.916275935658585 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.866399671274151 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.859316989447514 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.859241384919615 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.595744973057946 \leftarrow \text{Detail_1_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#17} = \begin{pmatrix} 998.785869565217 \leftarrow \text{Engine_part_1} \\ 998.101086956522 \leftarrow \text{Engine_part_2} \\ 997.215217391304 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#18} = \begin{pmatrix} 2.07535544862551 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.66700697298192 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.08215744557726 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.758900804380914 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.7253631489345 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.68548372534194 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.634947184948142 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.370785269209812 \leftarrow \text{Detail_8_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#18} = \begin{pmatrix} 997.670423497268 \leftarrow \text{Engine_part_1} \\ 997.654508196721 \leftarrow \text{Engine_part_2} \\ 997.594398907104 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#19} = \begin{pmatrix} 1.41935483870968 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.41935483870968 \leftarrow \text{Detail_5_Engine_part_2} \\ 1.41935483870968 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.41935483870968 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.41935483870968 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.64516129032258 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.129032258064516 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.129032258064516 \leftarrow \text{Detail_7_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#19} = \begin{pmatrix} 999.1 \leftarrow \text{Engine_part_3} \\ 999.1 \leftarrow \text{Engine_part_2} \\ 999.1 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#20} = \begin{pmatrix} 1.82638944863613 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.46492077338844 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.3127218776429 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.08441769793075 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.884669485678648 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.760997972443732 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.418555995350221 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.247326748929188 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#20} = \begin{pmatrix} 998.408333333333 \leftarrow \text{Engine_part_1} \\ 998.366666666667 \leftarrow \text{Engine_part_2} \\ 998.233333333333 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#21} = \begin{pmatrix} 1.5643331821676 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.44969821830549 \leftarrow \text{Detail_5_Engine_part_2} \\ 1.20062971116291 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.19272208222987 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.08599399543018 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.726223639433487 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.579966384383179 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.200432786887284 \leftarrow \text{Detail_2_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#21} = \begin{pmatrix} 998.706201550388 \leftarrow \text{Engine_part_1} \\ 998.582170542636 \leftarrow \text{Engine_part_2} \\ 998.55503875969 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#22} = \begin{pmatrix} 1.48595379814876 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.3992062901988 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.36306285831089 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.18378952062624 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.797761140648707 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.755847585503628 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.510097068026351 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.504281738536629 \leftarrow \text{Detail_1_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#22} = \begin{pmatrix} 998.936809815951 \leftarrow \text{Engine_part_1} \\ 998.906134969325 \leftarrow \text{Engine_part_2} \\ 998.643865030675 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#23} = \begin{pmatrix} 1.51974506103805 \leftarrow \text{Detail_2_Engine_part_1} \\ 1.49412758508085 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.23800034698001 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.16969170245139 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.802559118046229 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.776945966501693 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.751325955785186 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.247604264116591 \leftarrow \text{Detail_8_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#23} = \begin{pmatrix} 998.697604264117 \leftarrow \text{Engine_part_2} \\ 998.583333333333 \leftarrow \text{Engine_part_3} \\ 998.525 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#24} = \begin{pmatrix} 1.97440837056106 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.58313869699949 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.1316831792052 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.98119732120656 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.951090691079327 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.589913939634108 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.439433564839818 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.349134236474435 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#24} = \begin{pmatrix} 998.127272727273 \leftarrow \text{Engine_part_3} \\ 997.968181818182 \leftarrow \text{Engine_part_1} \\ 997.945454545455 \leftarrow \text{Engine_part_2} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#25} = \begin{pmatrix} 2.03063150371841 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.89135431980172 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.26461214917515 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.805017038844625 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.651803703326827 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.568282046517389 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.526461457599801 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.261837781016076 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#25} = \begin{pmatrix} 998.269565217391 \leftarrow \text{Engine_part_3} \\ 998.008695652174 \leftarrow \text{Engine_part_2} \\ 997.965217391304 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#26} = \begin{pmatrix} 1.84541186641454 \leftarrow \text{Detail_5_Engine_part_2} \\ 1.75938040818596 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.31210543102279 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.941946070817829 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.73203808963223 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.636357981451709 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.556241982870759 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.216518169604176 \leftarrow \text{Detail_8_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#26} = \begin{pmatrix} 998.408827238335 \leftarrow \text{Engine_part_1} \\ 998.233543505675 \leftarrow \text{Engine_part_2} \\ 998.1578814628 \leftarrow \text{Engine_part_3} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#27} = \begin{pmatrix} 1.25298664075795 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.14565111350448 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.03585952356623 \leftarrow \text{Detail_2_Engine_part_1} \\ 1.01118624131348 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.919389758151933 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.909515487637171 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.893907278743006 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.831503956325752 \leftarrow \text{Detail_3_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#27} = \begin{pmatrix} 999.308212765957 \leftarrow \text{Engine_part_3} \\ 999.051913060984 \leftarrow \text{Engine_part_1} \\ 998.563680851064 \leftarrow \text{Engine_part_2} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#28} = \begin{pmatrix} 1.61266571744888 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.41884462754784 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.38354081527214 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.1801149163528 \leftarrow \text{Detail_2_Engine_part_1} \\ 0.829932084943789 \leftarrow \text{Detail_4_Engine_part_1} \\ 0.591159027072826 \leftarrow \text{Detail_6_Engine_part_3} \\ 0.493058279330188 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.490684532031549 \leftarrow \text{Detail_5_Engine_part_2} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#28} = \begin{pmatrix} 998.89953746531 \leftarrow \text{Engine_part_3} \\ 998.787141535615 \leftarrow \text{Engine_part_2} \\ 998.046160962072 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#29} = \begin{pmatrix} 1.51724137931034 \leftarrow \text{Detail_1_Engine_part_1} \\ 1.17241379310345 \leftarrow \text{Detail_8_Engine_part_3} \\ 1.17241379310345 \leftarrow \text{Detail_6_Engine_part_3} \\ 1.17241379310345 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.17241379310345 \leftarrow \text{Detail_3_Engine_part_1} \\ 0.827586206896552 \leftarrow \text{Detail_7_Engine_part_3} \\ 0.827586206896552 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.137931034482759 \leftarrow \text{Detail_2_Engine_part_1} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#29} = \begin{pmatrix} 998.9 \leftarrow \text{Engine_part_3} \\ 998.9 \leftarrow \text{Engine_part_2} \\ 998.9 \leftarrow \text{Engine_part_1} \end{pmatrix}
\end{array}\right\};$$

$$\left.\begin{array}{l}
\text{Impact of details for parameter \#30} = \begin{pmatrix} 1.38619826401023 \leftarrow \text{Detail_4_Engine_part_1} \\ 1.34849312888255 \leftarrow \text{Detail_2_Engine_part_1} \\ 1.07207834239354 \leftarrow \text{Detail_7_Engine_part_3} \\ 1.05798377194 \leftarrow \text{Detail_3_Engine_part_1} \\ 1.00118370996565 \leftarrow \text{Detail_5_Engine_part_2} \\ 0.872505955626895 \leftarrow \text{Detail_8_Engine_part_3} \\ 0.785764164438642 \leftarrow \text{Detail_1_Engine_part_1} \\ 0.475792662742498 \leftarrow \text{Detail_6_Engine_part_3} \end{pmatrix} \\
\text{Impact of engine parts for parameter \#30} = \begin{pmatrix} 999.12304760463 \leftarrow \text{Engine_part_3} \\ 999.04024864244 \leftarrow \text{Engine_part_1} \\ 998.748547738606 \leftarrow \text{Engine_part_2} \end{pmatrix}
\end{array}\right\}.$$

Remark to [E]. The influences of details of a hydrogen power device for each of the 30 parameters were also found by us using one of the most powerful methods in the theory of expert assessments – the Analytical Hierarchy Process developed by American mathematician Thomas L. Saaty in the 1970s (Saaty, 1994; 1991; Saaty & Vargas, 1984). The application of this method gave results consonant with the above calculations, obtained using the approach described in Guseynov et al. (2015). Below are the obtained expert assessments, where

– D*i meens the engine detail* $\#i,\ i=\overline{1,8}$;

– *"j" means the expert's score (j=1 is the minimum score, j=8 is the maximum score).*

Expert evaluation of impact by AHP =

SCORES

"8"	"7"	"6"	"5"	"4"	"3"	"2"	"1"
D5	D3	D4	D1	D6	D7	D2	D8
D4	D2	D6	D1	D3	D8	D7	D5
D3	D8	D7	D2	D1	D4	D5	D6
D1	D3	D4	D2	D5	D7	D6	D8
D5	D1	D2	D8	D3	D4	D7	D6
D8	D3	D4	D2	D7	D1	D5	D6
D8	D3	D4	D7	D2	D1	D5	D6
D8	D7	D3	D4	D2	D1	D6	D5
D7	D8	D4	D3	D2	D1	D6	D5
D8	D3	D4	D2	D7	D1	D5	D6
D3	D8	D5	D4	D2	D7	D6	D1
D7	D8	D6	D1	D4	D2	D5	D3
D8	D3	D7	D2	D4	D5	D1	D6
D5	D7	D4	D1	D3	D6	D8	D2
D8	D3	D7	D4	D2	D5	D1	D6
D3	D5	D8	D4	D1	D7	D2	D6
D6	D5	D4	D2	D3	D7	D8	D1
D3	D4	D5	D2	D1	D6	D7	D8
D6	D5	D4	D3	D2	D1	D8	D7
D8	D7	D4	D1	D3	D5	D2	D6
D6	D5	D3	D4	D7	D1	D8	D2
D4	D3	D7	D5	D6	D8	D2	D1
D2	D7	D6	D1	D5	D4	D3	D8
D8	D4	D3	D7	D2	D5	D1	D6
D3	D7	D8	D5	D4	D1	D2	D6
D5	D4	D3	D7	D2	D6	D1	D8
D7	D8	D2	D1	D6	D5	D4	D3
D8	D7	D3	D2	D4	D6	D1	D5
D1	D8	D6	D4	D3	D7	D5	D2
D4	D2	D7	D3	D5	D8	D1	D6

PARAMETERS

DETAILS

$$\text{Expert evaluation of weight by AHP} = \begin{pmatrix} 4.694 & \text{is the expert evaluation for D3} \\ 4.500 & \text{is the expert evaluation for D4} \\ 4.250 & \text{is the expert evaluation for D8} \\ 4.111 & \text{is the expert evaluation for D7} \\ 3.472 & \text{is the expert evaluation for D5} \\ 3.389 & \text{is the expert evaluation for D2} \\ 3.028 & \text{is the expert evaluation for D1} \\ 2.556 & \text{is the expert evaluation for D6} \end{pmatrix}.$$

Using the above shown numerical calculations (values of weighted coefficients, influence coefficients, excess air coefficient, stoichiometric coefficient, etc.), the necessary initial data from Heywood (2018); Teh et al. (2008); Krutov (1989) as well as the following calculation formulas (35)-(37) (Heywood, 2018; Teh *et al.*, 2008; Krutov, 1989), numerical experiments were carried out to determine the degree of forcing and the effective engine power of a model hydrogen power device.

The law of piston motion in the considered model hydrogen power device:

$$U_{\text{engine\_piston}}(t) = \frac{\pi}{2} \cdot U_0 \cdot \sin(2\cdot\pi\cdot\nu\cdot t) \cdot \left(1 + \frac{\cos(2\cdot\pi\cdot\nu\cdot t)}{\sqrt{R^2 - \sin^2(2\cdot\pi\cdot\nu\cdot t)}}\right), \tag{35}$$

where U_0 (m/s) there is a value that depends on the piston stroke and the angular velocity of the crankshaft; ν (s^{-1}) is the angular velocity of the crankshaft; R is the ratio of connecting rod length to crank radius.

Dynamics of the indicator engine power of the considered model hydrogen power device:

$$\begin{aligned} I_{\text{indicated\_power}}(t) &= \pi \cdot D^2_{\text{piston\_diameter}} \cdot V_{\text{useful\_capacity\_in\_litres}} \cdot N_{\text{number\_cylinder}} \times \\ &\times \frac{M_{\text{piston\_stroke}}\left(U_{\text{engine\_piston}}(t)\right) \cdot P_{\text{indicated\_power}}(t) \cdot R_{\text{crankshaft}}(t)}{4 \cdot \chi_{\text{engine\_characteristic\_number}}}, \end{aligned} \tag{36}$$

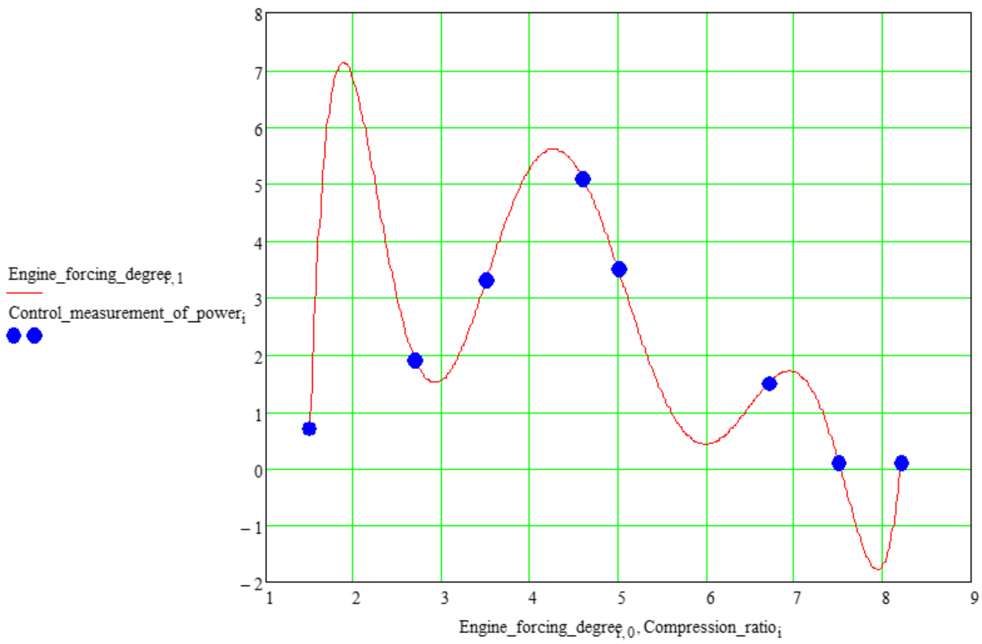
where $D_{\text{piston\_diameter}}$ is the piston diameter; $V_{\text{useful\_capacity\_in\_litres}}$ is the sum of the working volumes of all cylinders; $N_{\text{number\_cylinder}}$ is the number of cylinders; $M_{\text{piston\_stroke}}$ the piston stroke; $P_{\text{indicated\_power}}$ is the indicator pressure; $R_{\text{crankshaft}}$ is the number of revolutions of the crankshaft per minute; $\chi^{\text{indicated\_power}}_{\text{engine\_characteristic\_number}}$ is one of the characteristic numbers of the considered hydrogen power device, the value of which depends on the engine cycle (during numerical experiments, following Heywood, 2018, the value of this parameter was taken equal to 225).

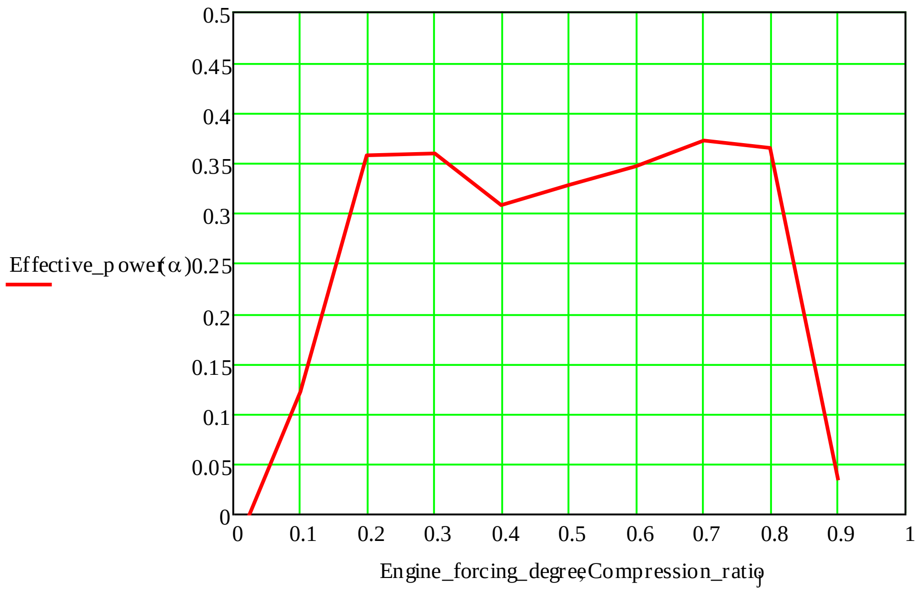
Dynamics of the effective power of engine of the considered model hydrogen power device:

$$E_{\text{effective\_power}}(t) = \frac{\vec{M}_{\text{torsional\_moment}}(t) \cdot R_{\text{crankshaft}}(t)}{\varphi_{\text{engine\_characteristic\_number}}}, \tag{37}$$

where $\vec{M}_{\text{torsional\_moment}}$ is a vector quantity characterizing the engine torque of the considered hydrogen power device, equal to the circumferential force on the flywheel per flywheel radius; $\varphi_{\text{engine\_characteristic\_number}}$ is also one of the characteristic numbers of the considered hydrogen power device, the value of which also depends on the engine cycle (during numerical experiments, following Heywood (2018), the value of this parameter was taken equal to 716.2).

The obtained numerical results of the experiments carried out are reflected in the following two graphs (both graphs characterize only one life cycle of the considered model power device without any additional loads):





3. Conclusion

In this work, physico-mathematical models have been developed to study the life cycle of a single-cycle two-stroke gas engine operating on hydrogen technology. The work pays special attention to the issue of dependence and interaction between the basic kinetic and dynamic parameters that characterize at least one full life cycle of stable operation of a single-cycle two-stroke gas/stream engine of the Brown type: the basic kinetic and dynamic parameters are determined as well as the driven- and governing ones are identified among them; the nature of the relationship between the driven- and governing parameters of a Brown type engine is determined; the weighting coefficients of these parameters are found (without knowledge of these weights it is impossible to find out how effective the operating mode of the power plant of a Brown type engine will be, since there is a dependence between the efficiency of the power plant and the weights of the basic physical and gas-dynamic parameters). In addition, in this work, some calculation formulas are obtained, which make it possible to: (a) control the mass flow of the hydrogen-air mixture, in particular, determine the optimal composition of the hydrogen-air mixture; (b) establish the optimal dependence of the stoichiometric volume ratio on the excess air coefficient; etc.

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